2.53 ANALYSIS OF MAGNETIC ANOMALY IN FERRO-MAGNETIC LAMINAE

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SYNOPSIS

Magnetic flux distribution across the magnetic lamina section is changed by magnetizing force and its frequency, then, magnitude and distribution of magnetic permeability change according strength of magnetizing force as well as its frequency. Therefore, an anylitical method of estimating magnetization losses and magnetic flux distribution of the laminae due to magnetic hysteresis and eddy current is complicate and difficult.

In this article it is assumed that there is no higher harmonics but only a fundamental wave, magnitude and phase angle of the permeability of the lamina are changable according to magnetization force, frequency and distance from the center of the lamina, then we can calculate and analyze these complicated problems.

INTRODUCTION

When calculating eddy current in a ferromagnetic system, it is usual to assume that the permeability does not change with frequency, and is the same at all points in the system. In real case, distribution of magnetic fluxes is varied with frequency and it induces variation of permeability in the lamina section.

Therefore, there is the discrepancies which exist between results obtained experimentally and those determined from the classical solution in which permeability is considered as constant when the specimen is subjected to alternating current magnetization. In spite of various extensive literature[(1)~(4)] dealing with the discrepancies, the interaction between the eddy current, magnetic hysteresis and non linear relation between the magnetic flux density and exciting current prevent any strict quantitative analysis in any case. No worker has seemed to calculate the eddy current and the iron losses in the magnetic laminated core considering nonlinear relation between magnetic flux density and magnetizing force. In this paper, at first variable permeabibty is calculated approximately and using conception of complex permeability it is expressed by an exponential function of distance from center, frequency, and dimension of the specimen, after that, relation between the magnetic fluxes and magnetizing force are expressed by Bessels' equation of complex variables, and solving the equation we can calculate distribution of the fluxes and equivalent impedance of a coil containing magnetic laminated core.

FUNDAMENTAL EQUATION

In magnetic laminae, Maxwell's equations turn out following equation,

$$\nabla^2 H = \frac{1}{\rho}\frac{\partial B}{\partial t} \qquad \cdots\cdots(1)$$

where H is magnetic field strength, B magnetic flux density, ρ resistivity of lamina and t is time.

Fig. 1 shows the coordinate of the lamina. If we assume that there is no change of magnetic fluxes in v direction, equation (1) is reduced as follow,

$$\frac{\partial^2 H}{\partial u^2} + \frac{\partial^2 H}{\partial w^2} = \frac{1}{\rho}\frac{\partial B}{\partial t} \qquad \cdots\cdots(2)$$

Moreover, now we assume that flow directions of eddy current induced by change of magnetic fluxes is parallel to rims of the lamina, then we can put $u = ax$, $w = bx$, equation (2) turns out as follow,

$$\frac{\partial^2 H}{\partial x^2} = d^2 \frac{1}{\rho}\frac{\partial B}{\partial t} \qquad \cdots\cdots(3)$$

where

$$d^2 = \frac{a^2 b^2}{a^2 + b^2}$$

and a is a half length of thickness and b is a half length of breadth as shown in Fig. 2.

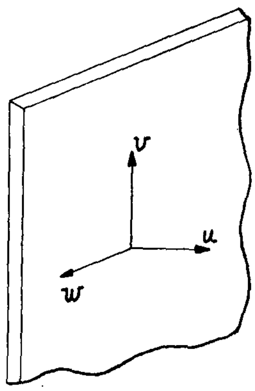

Fig. 1 Coordinate of the lamina, u axis represents thickness, v axis flow direction and w axis breadth of the lamina.

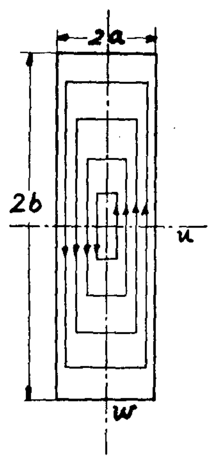

Fig. 2 Eddy current flows in parallel directions of rims of the lamina.

VARIABLE PERMEABILITY

Even though the sheet is homogeneous, it does not follow that we can assume the permeability is constant. When the laminae are subjected to magnetization in the direction of their length, shielding effect or skin effect of the eddy current results variation of magnetizating force which decreases across the lamina from the surface to the center of it. In practice, it is known that permeability changes according to magnetizing force or magnetic flux density.

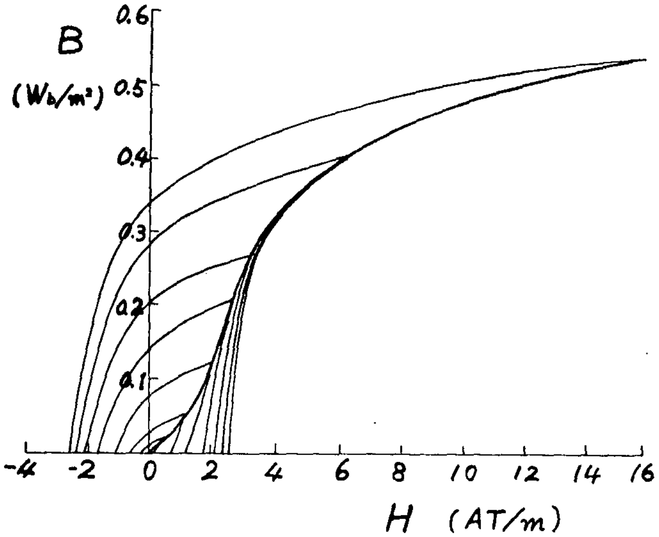

Fig 3, Magnetizing curve and hysteresis curves of 78.5 Permalloy.

For a given material, there is a definite magnetization curve and many hysteresis curves relating magnetizing force and magnetic flux density, for example, Fig. 3 shows these curves of 78.5 Permalloy. We can make equivalent ellipses whose areas are equal to the corresponding hysteresis loops as shown Fig. 4. From the equivalent ellipses we can find out phase difference angle θ between the magnetic field and magnetic flux density when the magnetic core is subjected in alternating current,

$$\theta=\sin^{-1}\frac{A}{\pi H_m B_m},\quad \dot{B}=\dot{\mu}\dot{H}=\mu e^{-j\theta}\dot{H}\cdots(4)$$

where A is area of hysteresis loop and $\dot{\mu}$ is complex permeability of the core.

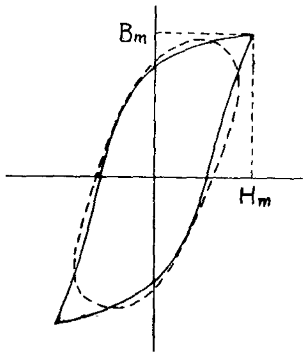

Fig. 4 A hysteresis loop and equivalent ellipse.

At first, we assume that the permeability is constant as usual, then equation (3) turns out as follow.

$$\frac{d^2\dot{H}}{dx^2}=j\frac{\omega d^2}{\rho}\dot{B}=j\frac{\omega\dot{\mu}d^2}{\rho}\dot{H}\cdots\cdots(5)$$

We can solve eqnation (5) in a boundary condition in which $\dot{H}$ is $\dot{H}_1$ when x is 1 and $\dot{H}$ is even function of x.

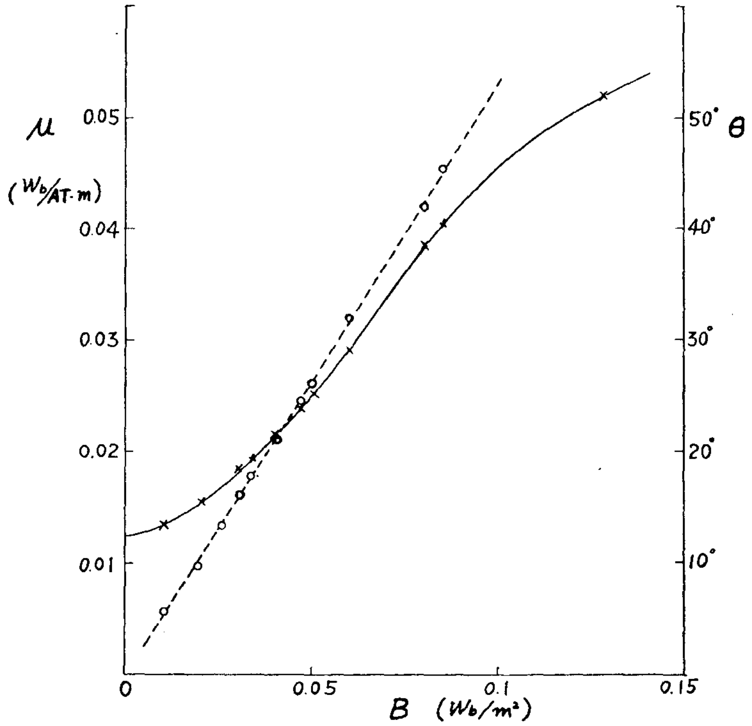

Fig. 5 Magnetic flux density vs complex permeability in which μ is effective value and θ is hysteresis angle of it.

$$\dot{H}=\frac{\dot{H}_1 \cosh \dot{\gamma} x}{\cosh \dot{\gamma}} \qquad \cdots\cdots (6)$$

where

$$\dot{\gamma}=\sqrt{\frac{j\,\omega\,\dot{\mu}}{\rho}}\,d=d\sqrt{\frac{\omega\,\mu\,|\,90^\circ-\theta}{\rho}}=m+j\,n$$

From above equation we have

$$\frac{B}{B_1}=\frac{H}{H_1}=\sqrt{\frac{\sinh^2 mx+\cos^2 nx}{\sinh^2 m+\cos^2 n}} \qquad \cdots\cdots (7)$$

Equation (7) shows the first aproach of distribution of magnetic flux density in lamina and B_1 is magnetic flux density at surface of the lamina and B_1 is found out from so called B-H curve and also the magnetic flux density B_0 at the center of the lamina is calculaled from equation (7) if we put x is zero.

Then from Fig. 5 we can decide complex permeability at the surface and the center of the lamina

$\mu=\mu_1 e^{-j\theta_1}$ when x is 1 at the surface

$\mu=\mu_0 e^{-j\theta_0}$ at the center of the lamina

CALCULATION OF DISTRIBUTION OF THE MAGNETIC FLUXES

From above mentioned conditions and calculations, we can consider the complex permeability is a fnnction of distance x from the center of the lamina, then we assume

$$\left.\begin{aligned}&\dot{\mu}=e^{2\dot{\alpha}+2\dot{\beta}x}\\&\dot{\alpha}=\alpha_1+j\,\alpha_2,\qquad \dot{\beta}=\beta_1+j\,\beta_2\end{aligned}\right\}\cdots\cdots(8)$$

and when x is zero

$$\mu_0=e^{2\alpha_1},\qquad -\theta_0=2\alpha_2\cdots\cdots(9)$$

when x is 1

$$\mu_1=e^{2\alpha_1+2\beta_1}\qquad -\theta_1=2(\alpha_2+\beta_2)\cdots\cdots(10)$$

and from eqnation (4) we can write

$$\frac{d^2H}{dx^2}=\frac{j\,\omega\,\dot{\mu}\,d^2}{\rho}\dot{H}=\gamma^2e^{2\dot{\alpha}+2\dot{\beta}x}\dot{H}\cdots\cdots(11)$$

Now we put

$$e^{\dot{\alpha}+\dot{\beta}x}=X\cdots\cdots(12)$$

then

$$\frac{dX}{dx}=\beta\,e^{\dot{\alpha}+\dot{\beta}x}$$

and equation (5) reforms as follow

$$\frac{d^2H}{dX^2}+\frac{1}{X}\frac{dH}{dX}+K^2H=0\cdots\cdots(13)$$

where

$$K^2=\frac{-\gamma^2}{\rho^2}=\frac{-j\,\omega\,d^2}{\beta^2\rho}$$

or

$$K=\frac{d}{\beta}\sqrt{\frac{\omega}{\rho}}\ \underline{\lfloor -450}\cdots\cdots(14)$$

Equation (13) is Bessel's equation of zero order and we can solve

$$H=C_1J_0(KX)+C_2Y_0(KX)\cdots\cdots(15)$$

C_2 in the equation (15) is zero because H is finite in every case and

$$H=H_1,\qquad X=e^{\dot{\alpha}+\dot{\beta}}=X_1\qquad \text{when } x=1$$

$$X=e^{\dot{\alpha}}=X_0\qquad \text{when } x=0$$

then we have

$$H=C_1J_0(KX)\cdots\cdots(16)$$

where

$$C_1=\frac{H_1}{J_0(KX_1)}\cdots\cdots(17)$$

and

$$\dot{B}=\dot{\mu}\dot{H}=X^2\dot{H}\cdots\cdots(18)$$

therefore, magnetic fluxes in the lamina becomes

$$\phi_1=4ab\int_0^1 B\,dx=4ab\int_0^1 C_1X^2J_0(KX)dx$$

$$=\frac{4ab\,C_1}{\beta}\int_{X_0}^{X_1} XJ_0(KX)dX$$

$$=\frac{4ab\,C_1}{\beta K}\{X_1J_1(KX_1)-X_0J_1(KX_0)\}\cdots\cdots(19)$$

Let us assume that cross sectional area of laminated core is S and magnetic fluxes of the core becomes

$$\phi=\frac{\sigma S\phi_1}{4ab}=\frac{\sigma S C_1}{\beta K}\{X_1J_1(KX_1)-X_0J_1(KX_0)\}$$

$$=\frac{\sigma S H_1 e^{\alpha}}{d\sqrt{\frac{\omega}{\rho}}\ \underline{|-45^\circ}\ J_0(z_1)}\{e^{\beta}J_1(z_1)-J_1(z_0)\}\cdots\cdots(20)$$

where

σ_1 is space factor of the core

$$z_1=\frac{d}{\beta}\sqrt{\frac{\omega}{\rho}}e^{\alpha+\beta}\ \underline{|-45^\circ}$$

$$z_0=\frac{d}{\beta}\sqrt{\frac{\omega}{\rho}}e^{\alpha}\ \underline{|-45^\circ}$$

From equdtion (20) we can calculate equivalent impedance $\dot{Z}$ of the coil containing magnetic laminated core.

$$\dot{Z}=\frac{j\omega n\phi}{i}=\frac{j\omega n}{i}I(\phi)+\frac{j\omega n}{i}R(\phi)\cdots\cdots(21)$$

where $I(\phi)$ is imaginary part of equation (20)

$R(\phi)$ is real part of equation (20)

EXAMPLES

We take laminated Permalloy ring type core as a specimen to ascertain the derived results are correct. The specimen has the following dimensions.

thickness : 0.35 mm. outerdiameter : 45 mm.

inner diameter : 33 mm. no. of sheet : 5. no. of turns : 300.

From these dimensions and curves of Figs 3 and 5, we can calculate following results:

In the case of 200 c/s.

We assume that exciting current is 0.8 mA that is seemed to be constant in any frequency. Then from the B–H curve showed in Fig. 3 we can take H_1 is 1.97 AT/m and from equation (6) $m \doteqdot n = 0.92$ $B_1 = 0.0474$ W_b/m^2, then

$$\alpha_1 = -1.926, \quad \beta_1 = 0.0621, \quad \dot{z}_1 = 17.95 \underline{|-26.2^\circ}$$

$$\alpha_2 = -0.177, \quad \beta_2 = -0.0375, \quad \dot{z}_0 = 16.87 \underline{|-24^\circ}$$

after these numeral values are calculated, we have the following last result

$$\dot{Z} = 274 + j\,407$$

In the other frequences

In similar process, we can calculate as follow

$f = 500$ c/s, $\dot{Z} = 605 + j\,596$

$f = 1000$ c/s, $\dot{Z} = 918 + j\,664$

$f = 3000$ c/s, $\dot{Z} = 1485 + j\,926$

$f = 10000$ c/s, $\dot{Z} = 3320 + j\,2080$

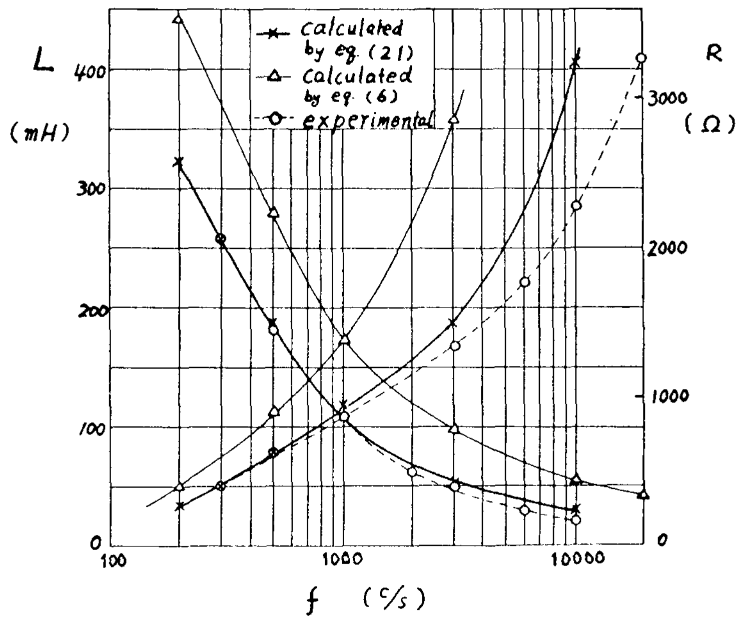

Fig. 6 Equivalent inductance L and resistance R and their comparison between calculated results and experimental results.

Fig. 6 shows these results and experimental results which are measured by impedance bridge, and also calculated results by obsolete method which is shown by equation (6) in which permeability is seemed to be constant.

Besides these sesults, we can calculate magnitic flux distribution in the lamina by equations (12), (16) and (18) but in this case we can not ascertain experimentally.

CONCLUTION

It would be possible for us to calculate correctly the distribution of magnetic fluxes in magnetic laminae and equivalent impedance of iron cored coil if we had measured basic characteristics of the laminated core, but it is a rather troublesome calculation because of using Bessel function of complex variables.[5]

References

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